Exam 1 Review, Mon 9/11/2023

Lon CAPA
Exam 1 Review
Lesson 6 #5

$$\int 12 \cot\left(\frac{x}{3}\right) dx$$

$$= \int 12 \frac{\cos\left(\frac{x}{3}\right)}{\sin\left(\frac{x}{3}\right)} dx = \int 12 \frac{3\,du}{u} = 36 \int \frac{1}{u} du$$

$u = \sin\left(\frac{x}{3}\right)$

$du = \cos\left(\frac{x}{3}\right) \cdot \frac{1}{3} dx$

$3du = \cos\left(\frac{x}{3}\right) dx$

$$= 36 \ln(|u|) + C$$

$$= 36 \ln\left(\left|\sin\left(\frac{x}{3}\right)\right|\right) + C$$

Fall 2022 Ex 1, #5

David is trying to make the perfect cup of tea. He, *somehow* knows how to pour the sugar at a rate of $s'(t) = \dfrac{t^3}{\sqrt{t^4+1}}$ grams per second into the tea. The tea will be perfect when there are exactly 10 grams of sugar. For how many seconds should David pour the sugar? **Note that there are initially zero grams of sugar in the tea.**

- 4.81 seconds
- 4.58 seconds
- 2.55 seconds
- 6.11 seconds
- 5.15 seconds
- 5.82 seconds

$s'(t)$ $\dfrac{\text{grams of sugar}}{\text{sec}}$

$s(0) = 0$

$s(t)$ grams of sugar

t = seconds

$$s(t) = \int s'(t)\,dt$$

$$s(t) = \int \frac{t^3}{\sqrt{t^4+1}}\,dt = \int \frac{\frac{1}{4}du}{\sqrt{u}} = \int \frac{1}{4} u^{-1/2}\,du$$

$u = t^4+1$

$du = 4t^3 dt$

$\frac{1}{4} du = t^3 dt$

$$= \frac{1}{4}\cdot 2 \cdot u^{1/2} + C$$

$$= \frac{1}{2}\sqrt{u} + C$$

$$= \frac{1}{2}\sqrt{t^4+1} + C$$

$s(0) = 0$ (x, $s(t)$)

So $s(t) = \frac{1}{2}\sqrt{t^4+1} + C$

$0 = \frac{1}{2}\sqrt{0^4+1} + C$

$0 = \frac{1}{2} + C$

$C = -\frac{1}{2}$

$s(t) = \frac{1}{2}\sqrt{t^4+1} - \frac{1}{2}$

When does cup have 10 grams?

$10 = \frac{1}{2}\sqrt{t^4+1} - \frac{1}{2}$

$$10.5 = \frac{1}{2}\sqrt{t^4+1}$$

$$21 = \sqrt{t^4+1}$$

$$21^2 = t^4+1$$

$$21^2-1 = t^4$$

$$440 = t^4$$

$$440 \ \boxed{2nd} \ \boxed{y^x} \ 4 \quad (\sqrt[x]{y})$$

OR

$$440 \ \boxed{y^x} \ (1/4)$$

$$t = \sqrt[4]{440} \approx 4.58$$

Spring 2023, Exam 1 #2

A faucet is turned on at 9:00am and water starts to flow into a tank at the rate of:

$$r'(t) = 12\sqrt{t},$$

where t is time in hours after 9:00am and the rate $r'(t)$ is in cubic feet per hour.

How many hours after 9:00am will there be **216** cubic feet of water in the tank?

- ○ 6
- ○ 18
- ○ 15
- ○ 9
- ○ 4.5
- ○ 13.5

$r'(t)$ $\frac{\text{cubic ft}}{\text{hr}}$

t – hours since 9 AM

$r(t)$ – cubic feet i.e. amount of H_2O in tank

Find t when $r(t) = 216$

$r(\underset{t=0}{0}) = \underset{r=0}{0}$

$$r(t) = \int r'(t)\,dt$$

$$r(t) = \int 12\sqrt{t}\,dt = \int 12\, t^{1/2}\,dt = 12 \cdot \frac{2}{3} t^{3/2} + C$$

$$= 8t^{3/2} + C$$

$$r(t) = 8t^{3/2} + C$$

use $r(0) = 0$ to find C

$$0 = 8(0)^{3/2} + C$$

$$0 = 0 + C$$

$$C = 0$$

$$r(t) = 8t^{3/2}$$

$$216 = 8t^{3/2}$$

$$27 = t^{3/2}$$

$$27^{2/3} = (t^{3/2})^{2/3}$$

$$9 = t$$

$27\ \boxed{y^x}\ (2/3)$

HW 7, #7

$\ln(x^y) = y\ln(x)$
$(\ln(x))^y$ - stuck
no log rule for this

$$\int 19x\,(\ln(4x))^2 dx.$$

Rd1:

$u = (\ln(4x))^2 \qquad dv = 19x$

$du = 2(\ln(4x))' \cdot \frac{1}{\cancel{4}x} \cdot \cancel{4}\,dx \qquad \int dv = \int 19x\,dx$

$du = 2\ln(4x) \cdot \frac{1}{x}\,dx \qquad v = \frac{19x^2}{2}$

$$\int 19x\,(\ln(4x))^2 dx = (\ln(4x))^2 \cdot \frac{19x^2}{2} - \int \frac{19x^{\cancel{2}}}{\cancel{2}} \cdot \cancel{2}\ln(4x) \cdot \frac{1}{\cancel{x}}\,dx$$

$$= \frac{19x^2}{2}(\ln(4x))^2 - \int 19x\ln(4x)\,dx$$

Rd2:

$u = \ln(4x) \qquad dv = 19x$

$du = \frac{1}{\cancel{4}x} \cdot \cancel{4}\,dx \qquad v = \frac{19x^2}{2}$

$$= \frac{19x^2}{2}(\ln(4x))^2 - \left[\frac{19x^2}{2}\ln(4x) - \int \frac{19}{2}x\,dx\right]$$

$$= \frac{19}{2}x^2(\ln(4x))^2 - \frac{19}{2}x^2\ln(4x) + \frac{19}{2}\frac{x^2}{2} + C$$